

# maths and algorithms behind photo-realistic graphics

dr. jon denning  
assistant professor of cse  
taylor university

frank s. brenneman lecture series

warning: there will be equations  
do not be afraid of them

goal of photo-realism: produce a computer-generated image that  
is indistinguishable from an actual photograph



[ bertil pinewood chair, 2006, [ikea link](#) ]

[ mikhalenko link ]



MIKHALENKO\_ART 2015





[valkyrie link]



[ valkyrie link ]

[ valkyrie link ]



VALKYRIE

NAUGHTY DOG



[ frank tzeng link ]

UNCHARTED 4  
*A Thief's End*



[ luc bégan link ]

[jungwon park link]



[ chris jones link ]



W.I.P.  
25/09/14  
[www.chrisj.com.au](http://www.chrisj.com.au)

[ andrew price link ]





[youyudetun, link]

the good

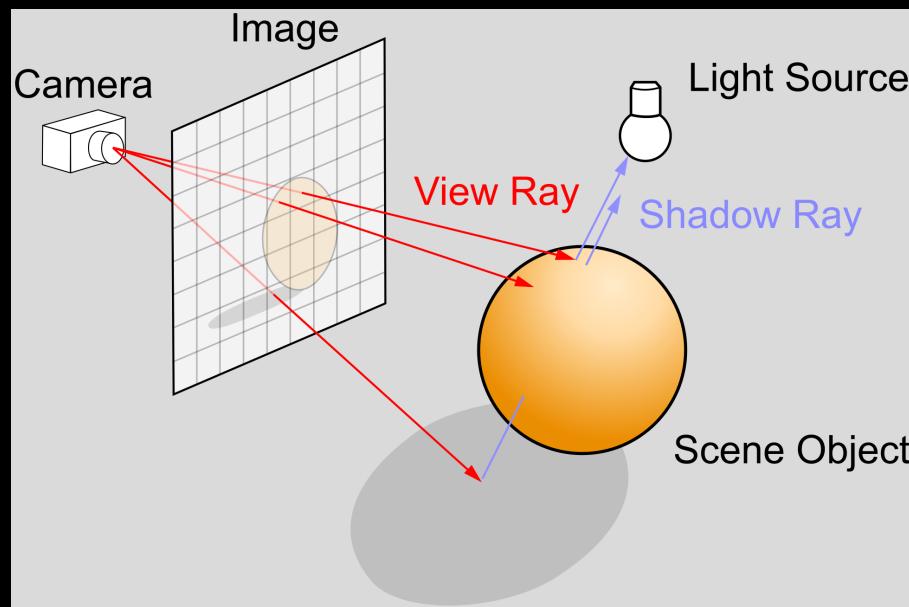
today we use the same principles behind the earliest cg



[ martin newell, link ]

## rendering system : light simulation

emit from source, enter camera, bounce around scene



modeling quality     $\Rightarrow$     render quality  
(light source, camera, materials, surfaces)

modeling light sources is relatively easy



[ paul debievec link ]

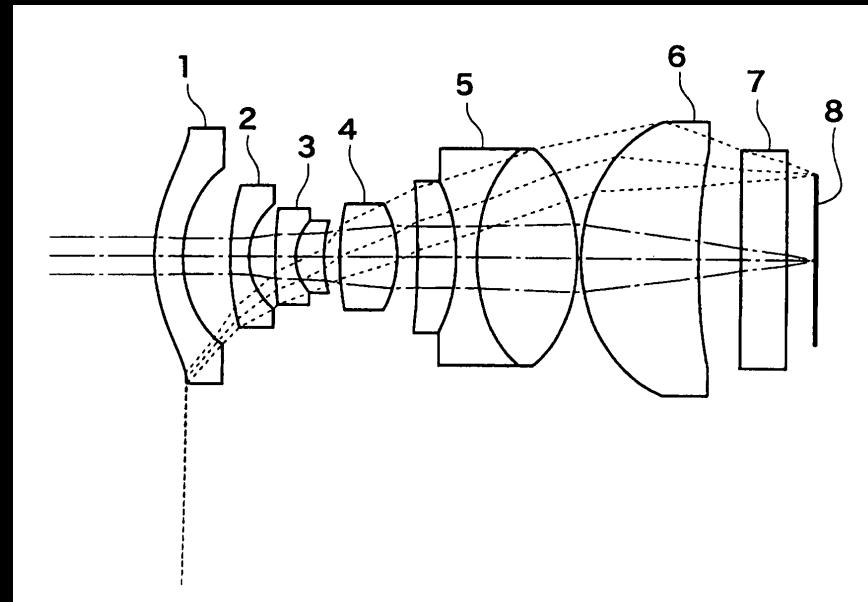


[oscar leif link, paul debevec link]

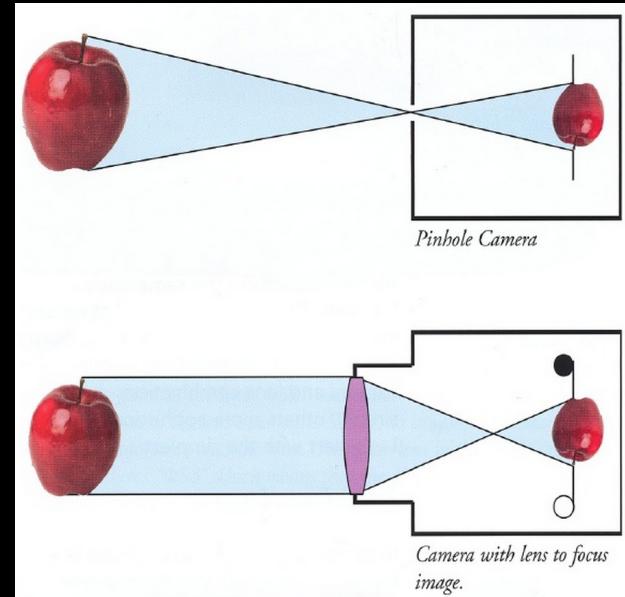
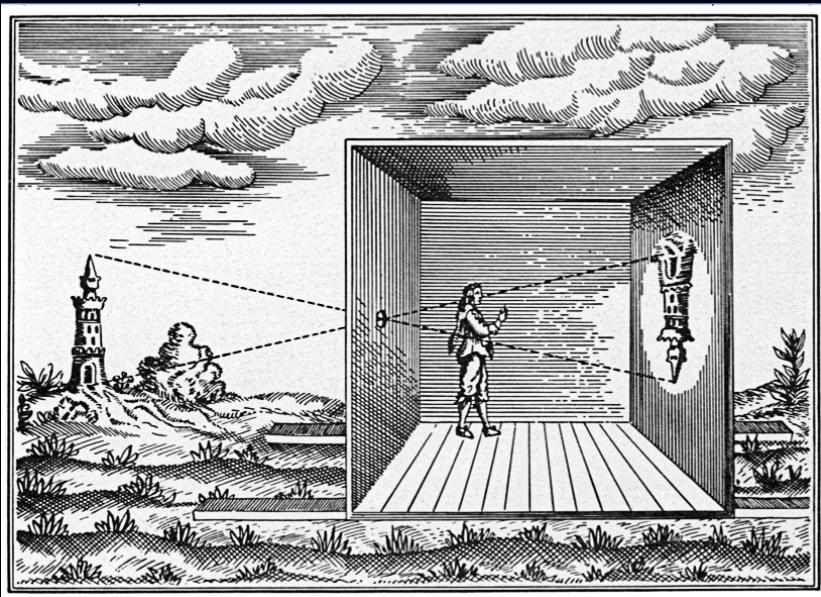


[ oscar leif link, paul debevec link ]

modeling camera lens system can be complex



pinhole works surprisingly well, much easier to control



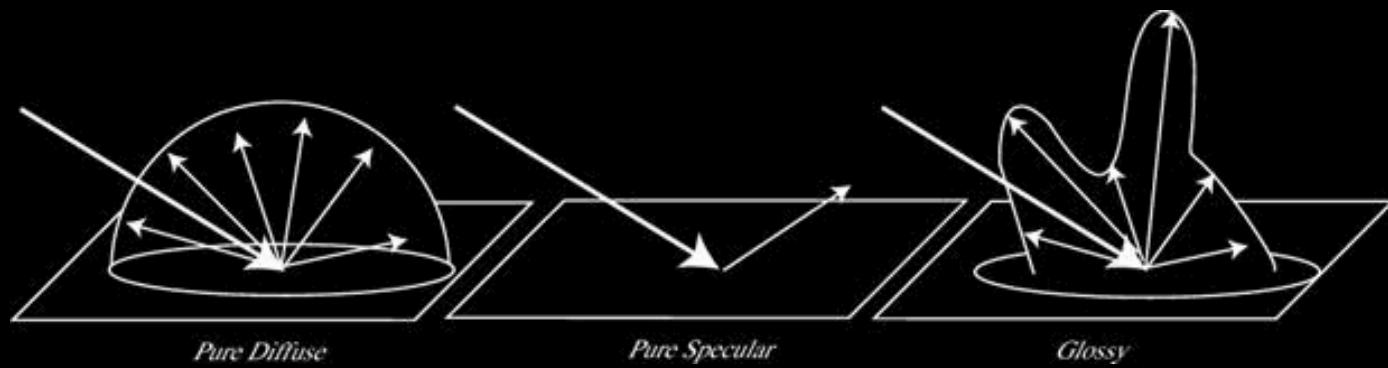
[ original author unknown link, john munno link ]

modeling materials is not difficult



[ marc tardif link ]

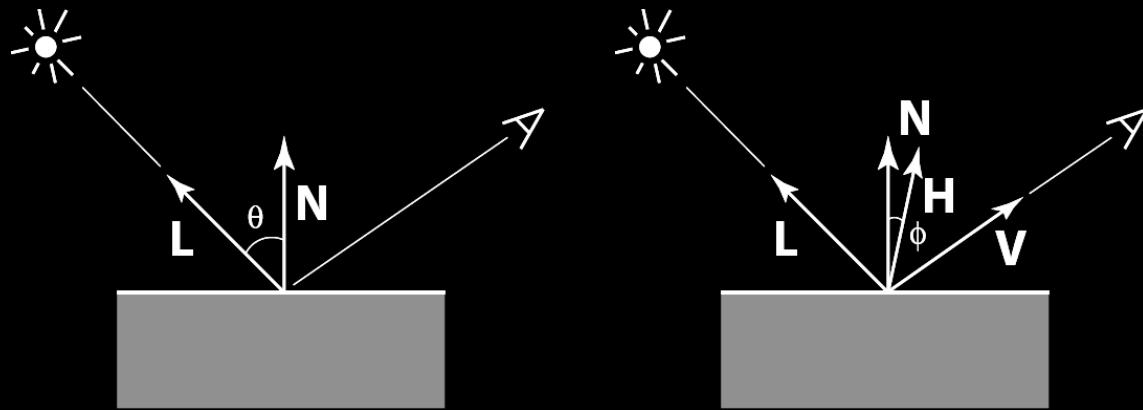
light reflects off surface in all directions at different amounts  
materials describe how much light bounces



we can faithfully describe almost any material as a  
math function ( $\rho$ ) with just a few parameters

diffuse  $\rho(\dots) = \frac{R_d}{\pi}$

specular  $\rho(\dots) = R_s (\mathbf{h} \cdot \mathbf{n})^p$



$$\begin{aligned}
\rho(\dots) &= a \cdot b \cdot F(\omega_i \cdot \mathbf{h}) \\
a &= \frac{\sqrt{(\textcolor{red}{n}_u + 1)(\textcolor{red}{n}_v + 1)}}{8\pi} \\
b &= \frac{(\mathbf{n} \cdot \mathbf{h})^{\textcolor{red}{n}_u} \cos^2 \phi + \textcolor{red}{n}_v \sin^2 \phi}{(\mathbf{h} \cdot \omega_i) \max(\cos \theta_i, \cos \theta_o)} \\
F(\omega_i \cdot \mathbf{h}) &= \textcolor{red}{R}_s + (1 - \textcolor{red}{R}_s)(1 - (\omega_i \cdot \mathbf{h}))^5
\end{aligned}$$

(schlick's approximation to the fresnel equation)

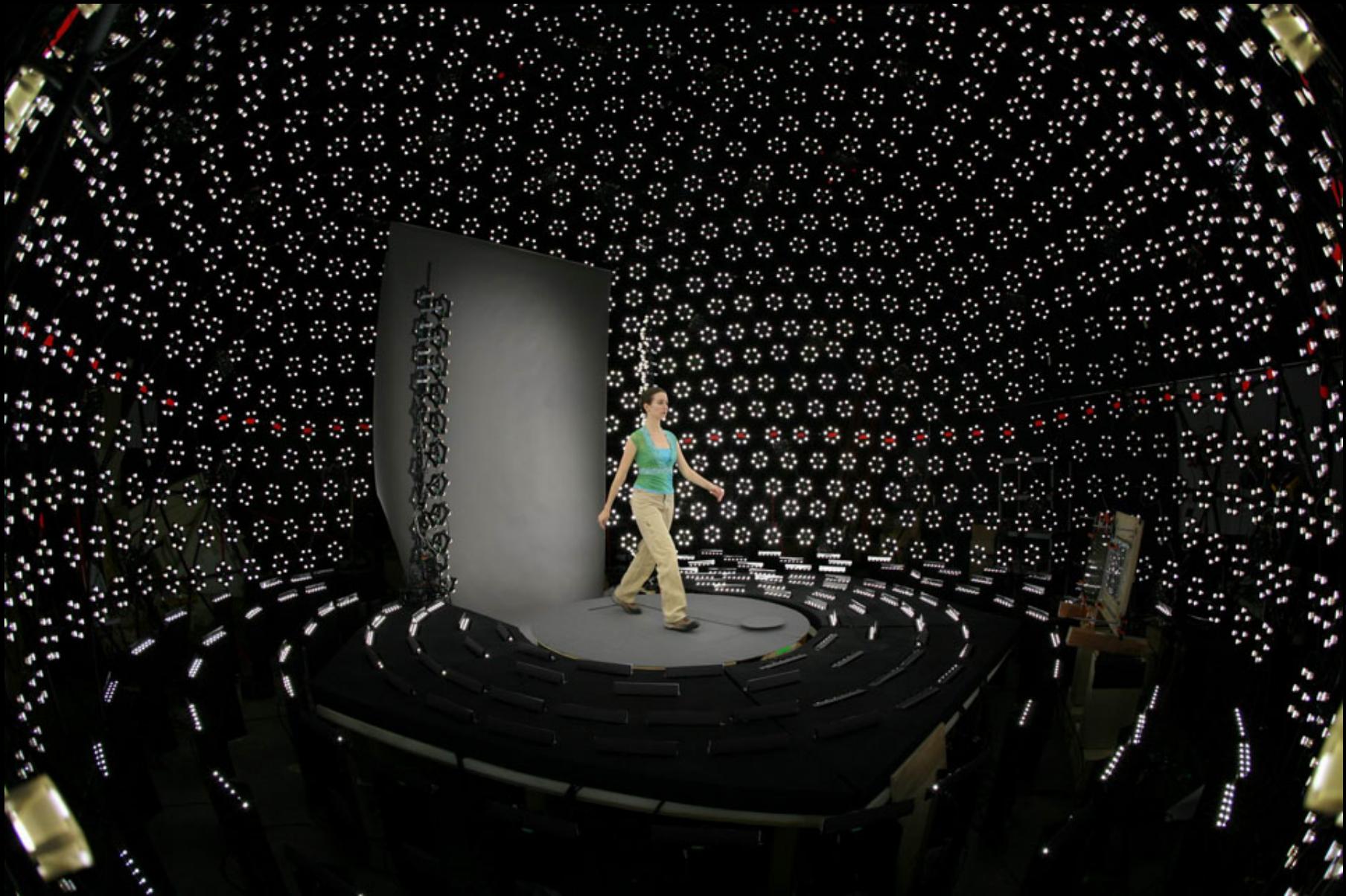


HENRIK WANN JENSEN - 2004

[jensen]

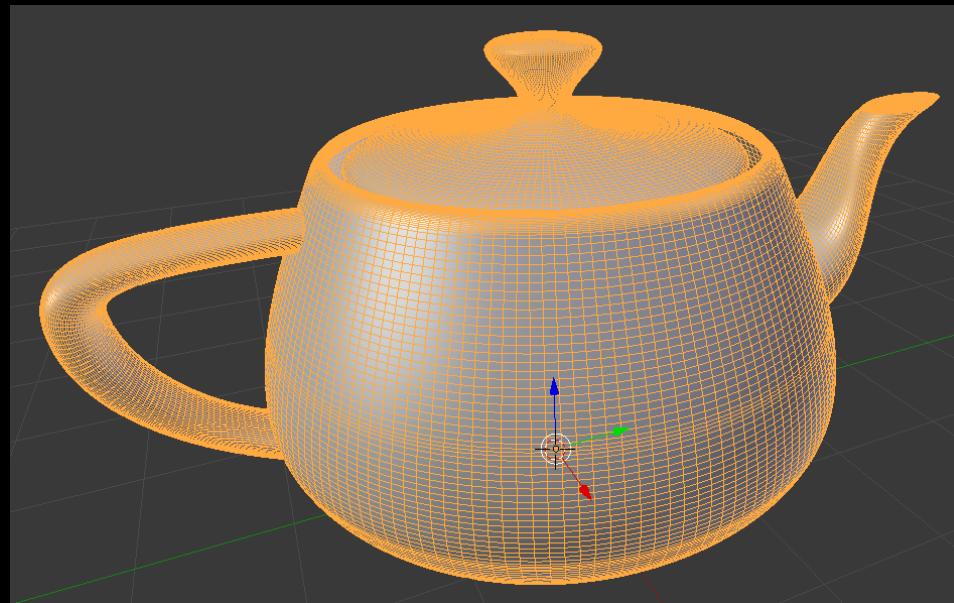


[jensen]



[debevec, link]

modeling surfaces is straightforward



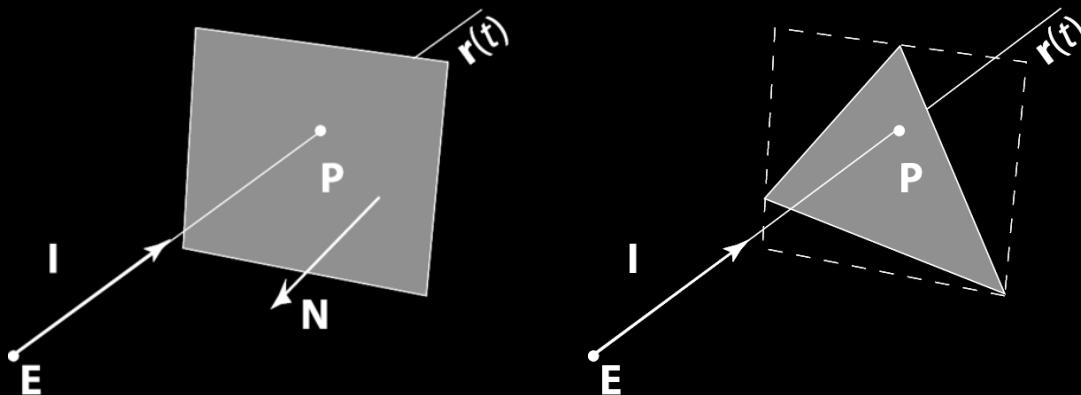
use triangles to describe any surface to arbitrary precision

using ray for light and triangle for surface,  
simulation involves simple algebra

ray  $P = P_o + tV$

plane  $P \cdot N + d = 0$

ray-plane  $t = -(P_o \cdot N + d)/(V \cdot N)$



the rendering equation puts the parts together  
all photo-realistic graphics engines solve this equation

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\Omega} \rho(\mathbf{x}, \omega_i, \omega_o) L_i(\mathbf{x}, \omega_i) (\omega_i \cdot \mathbf{n}) d\omega_i$$

emit ( $L_e$ ), camera ( $L_o$ ), bounce ( $\rho, L_i, L_o$ )

the only thing remaining is doing the light simulation  
solve the rendering equation

"what do we do now sheriff?"

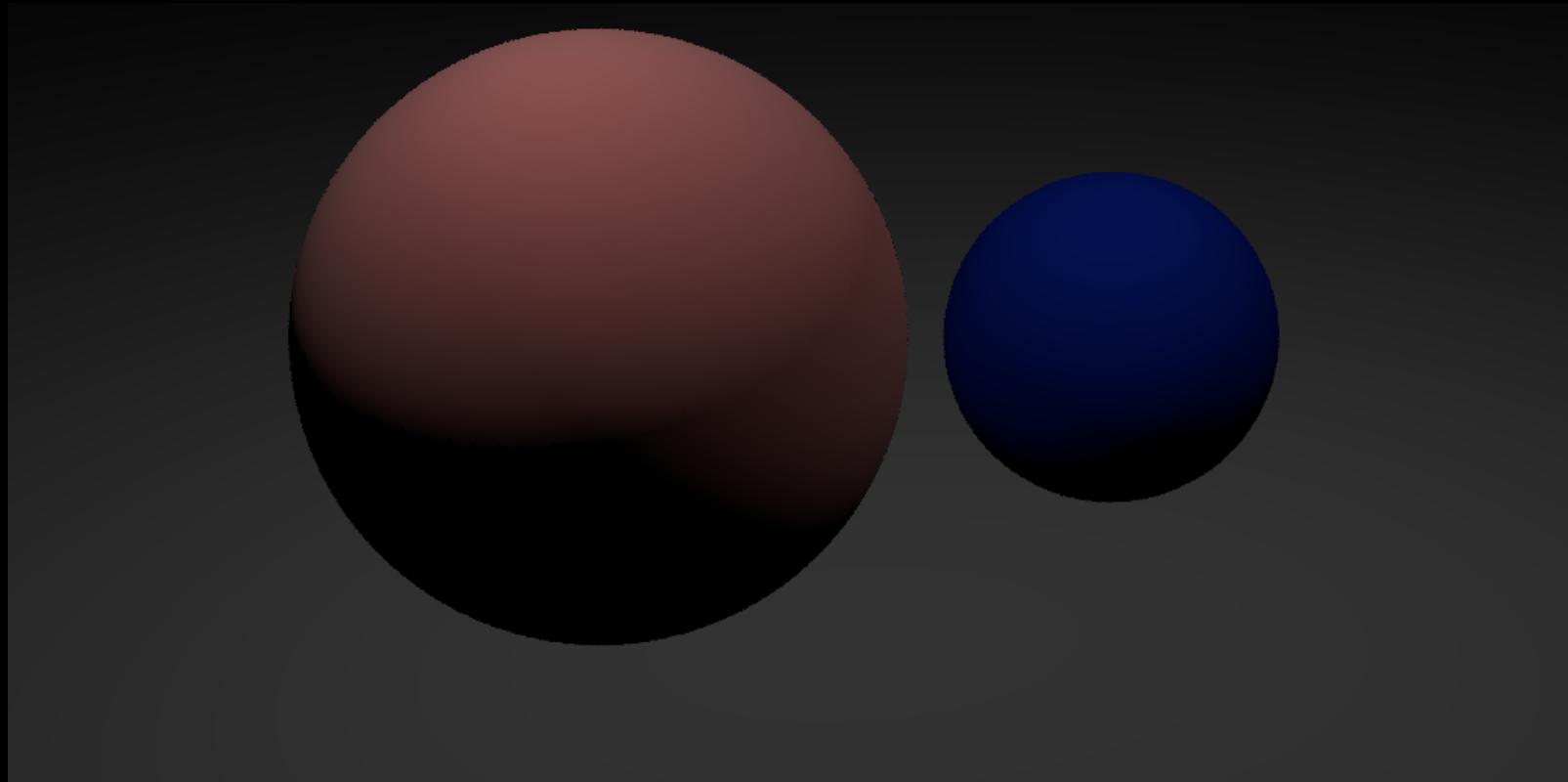
"now, we *render*"

the bad

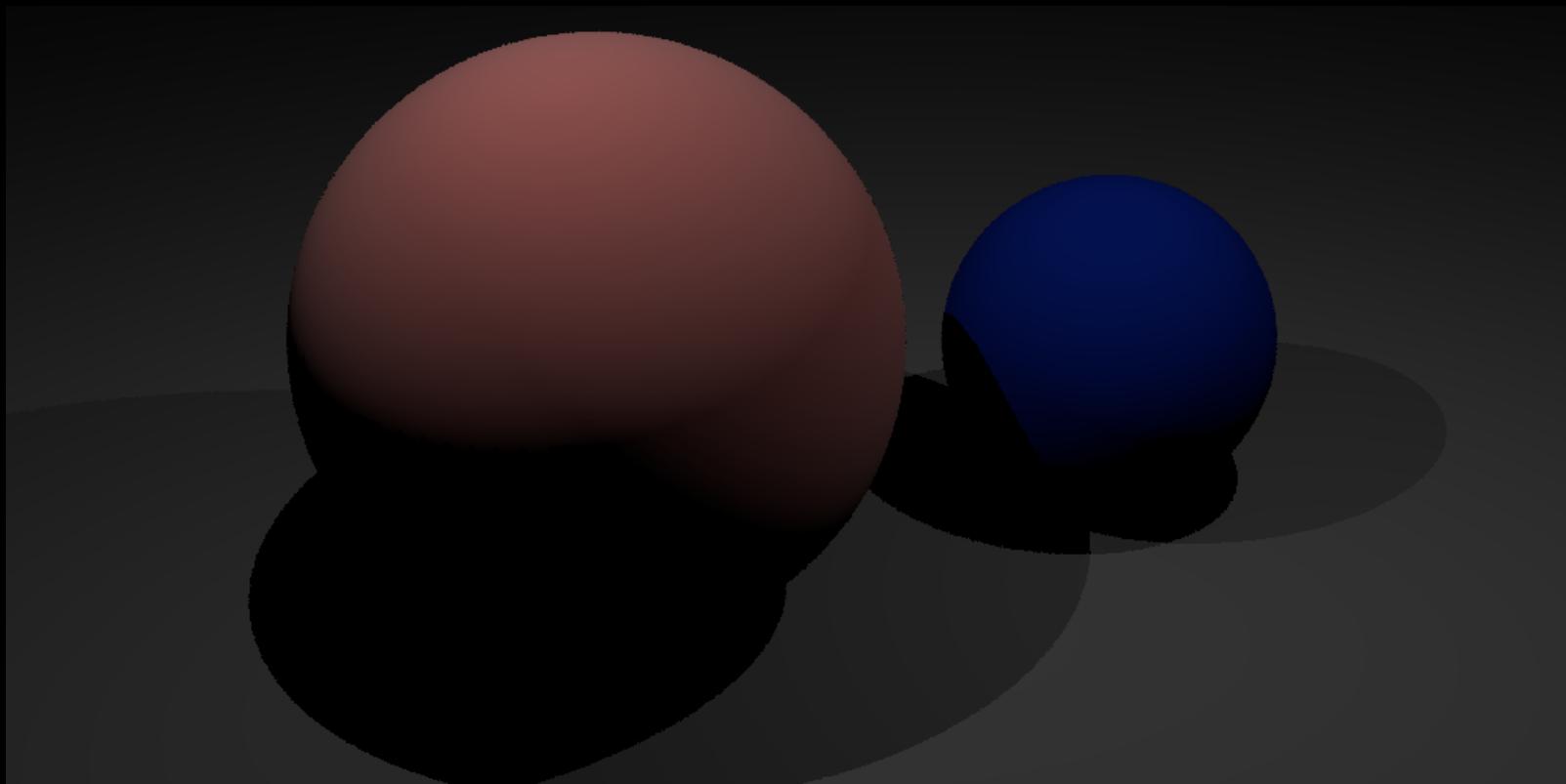
light simulation isn't cheap  
steps toward photo-realism causes computation time to grow

for example, let's look at rendering an image  
two spheres on a plane with two lights

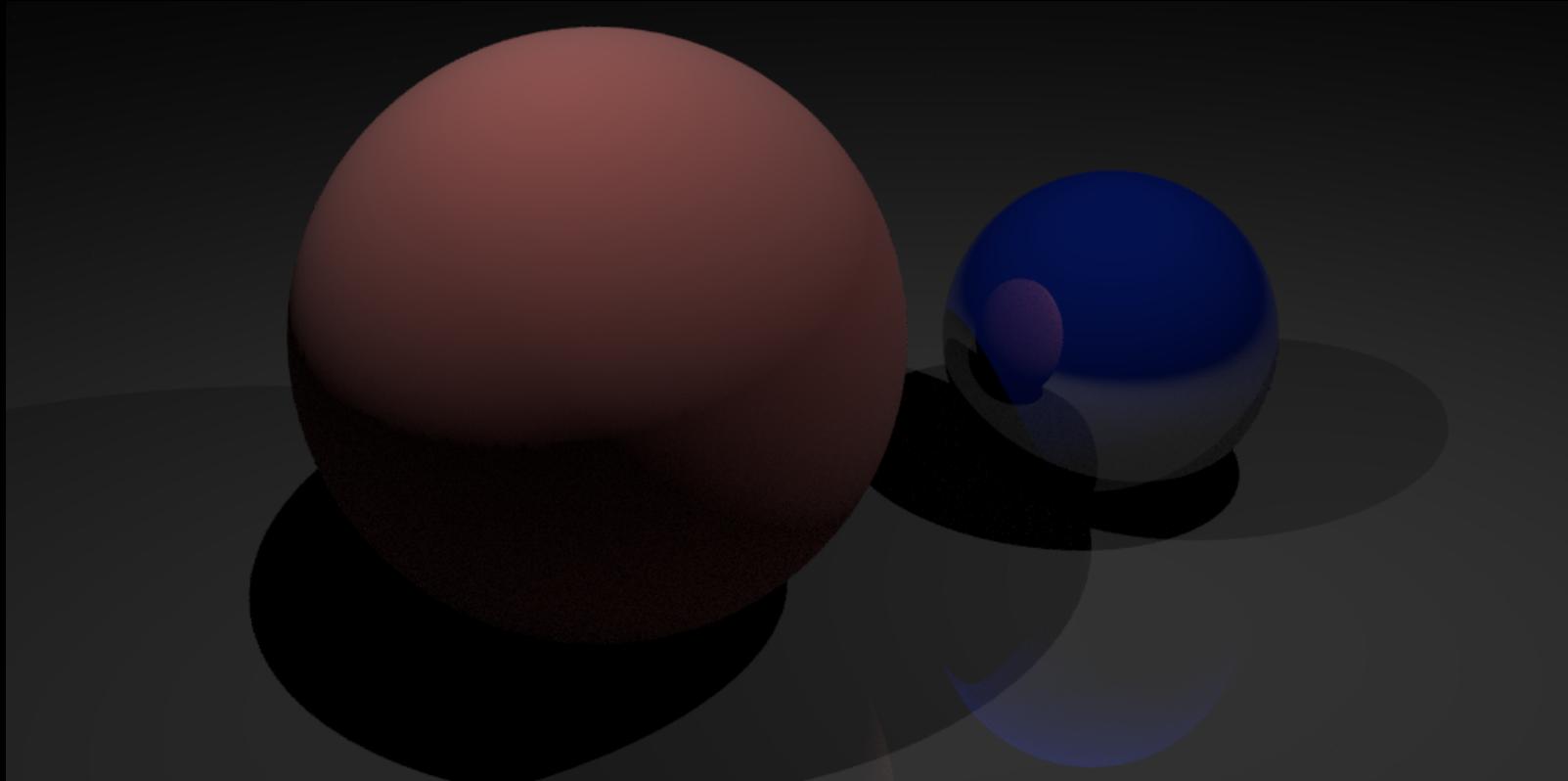
note: following simulations are already "smart"



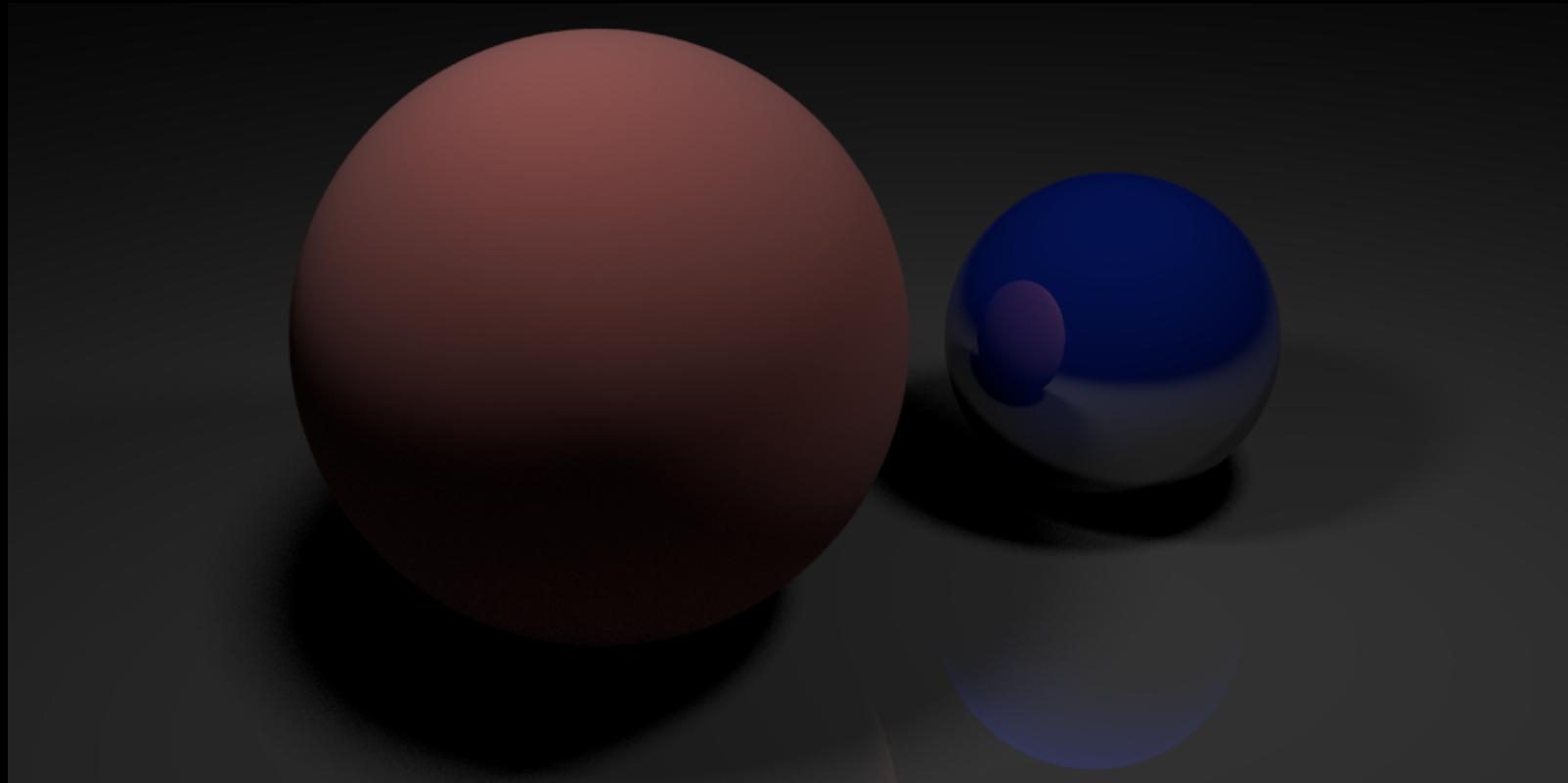
simple  
< 1 second



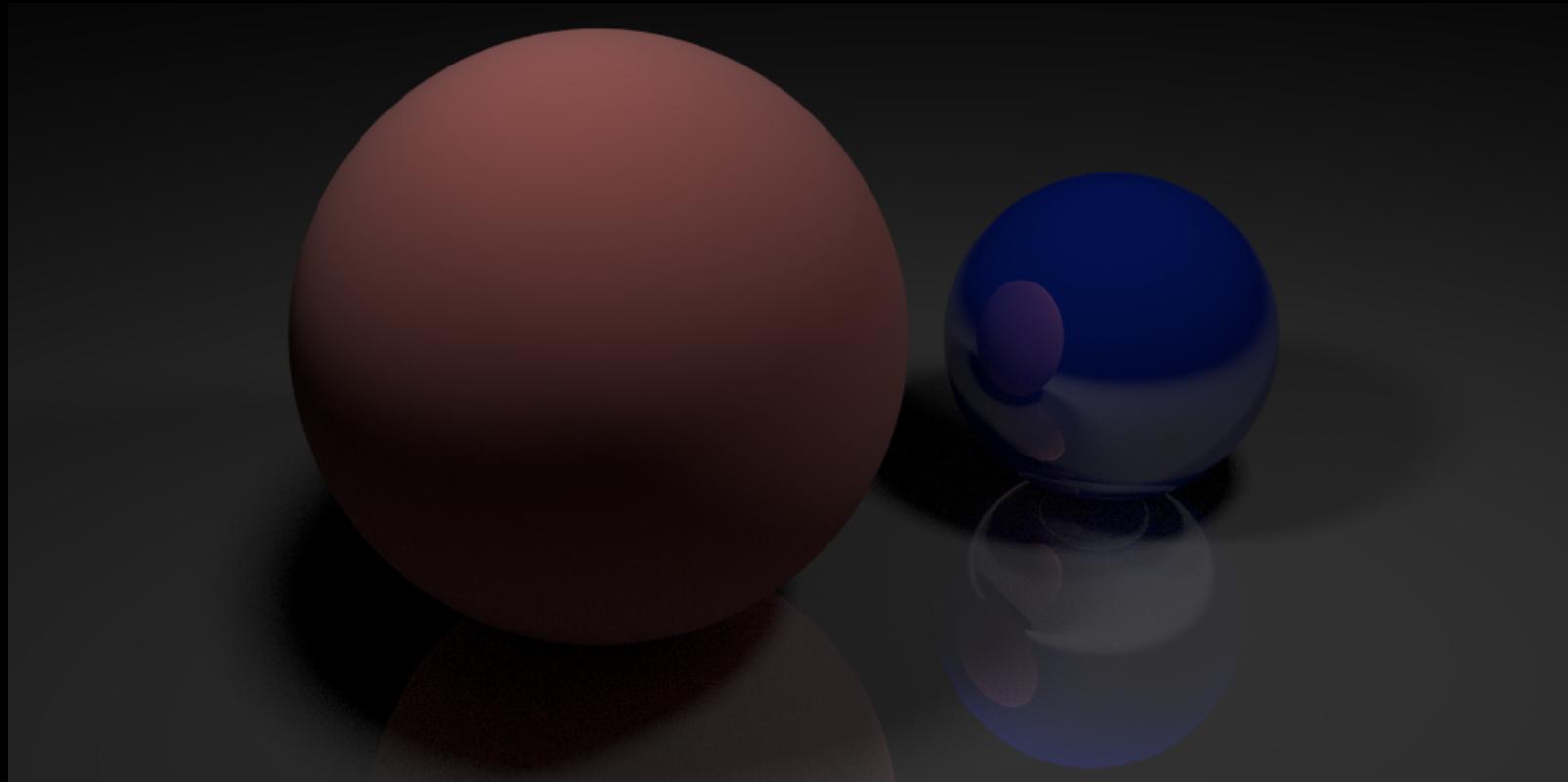
shadows  
1 second



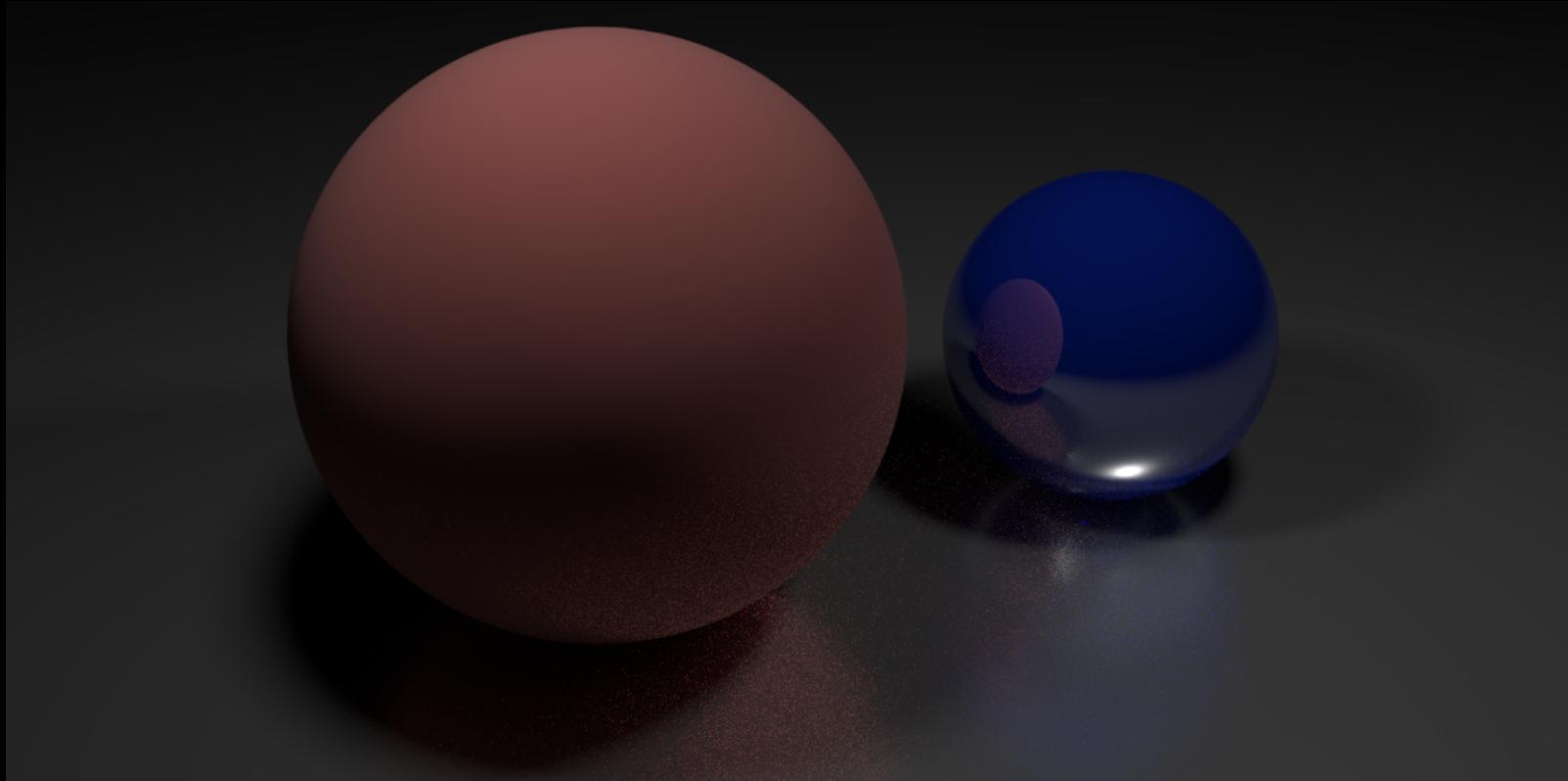
mirror reflection  
7 seconds



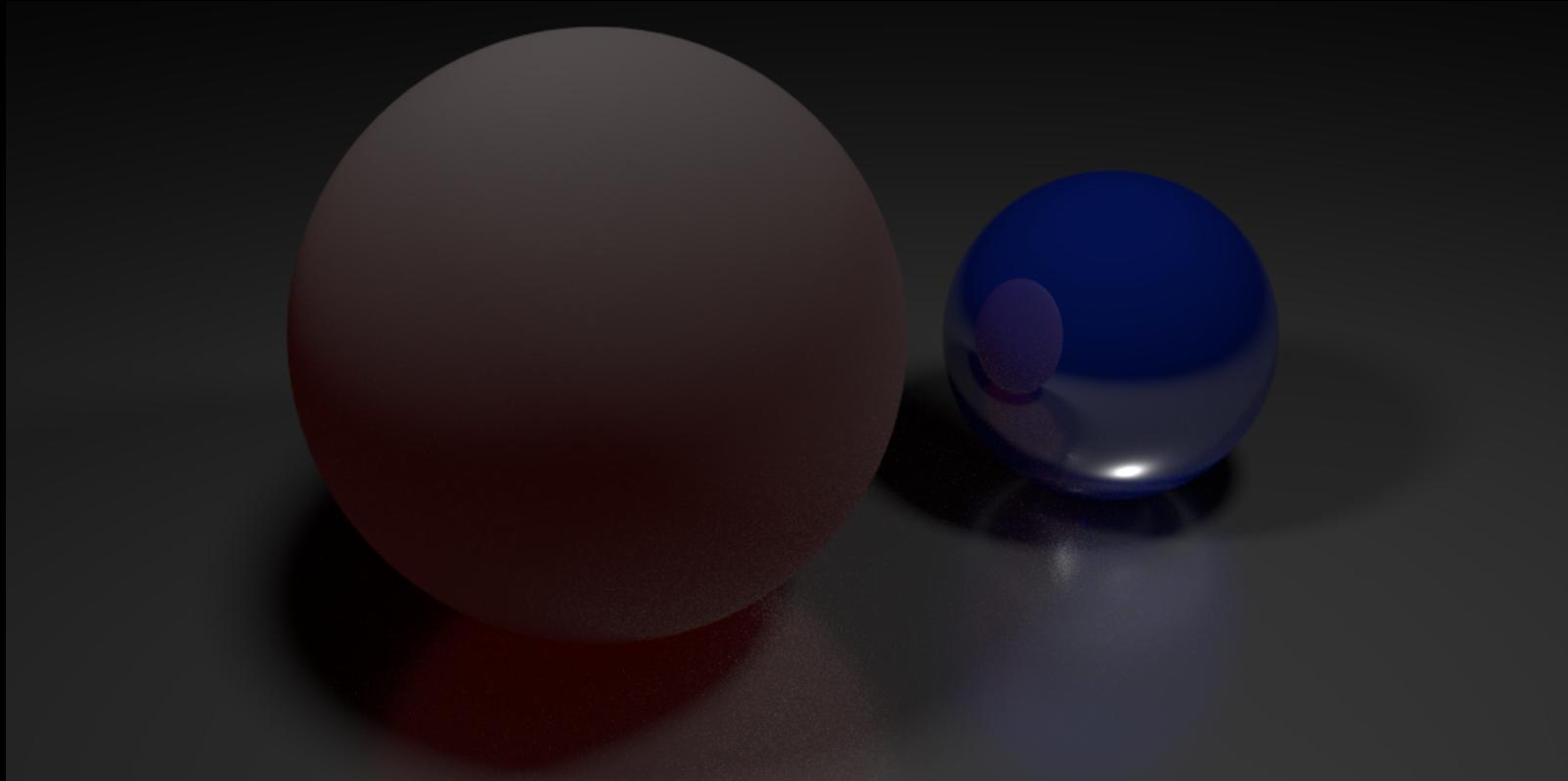
soft shadows  
30 seconds



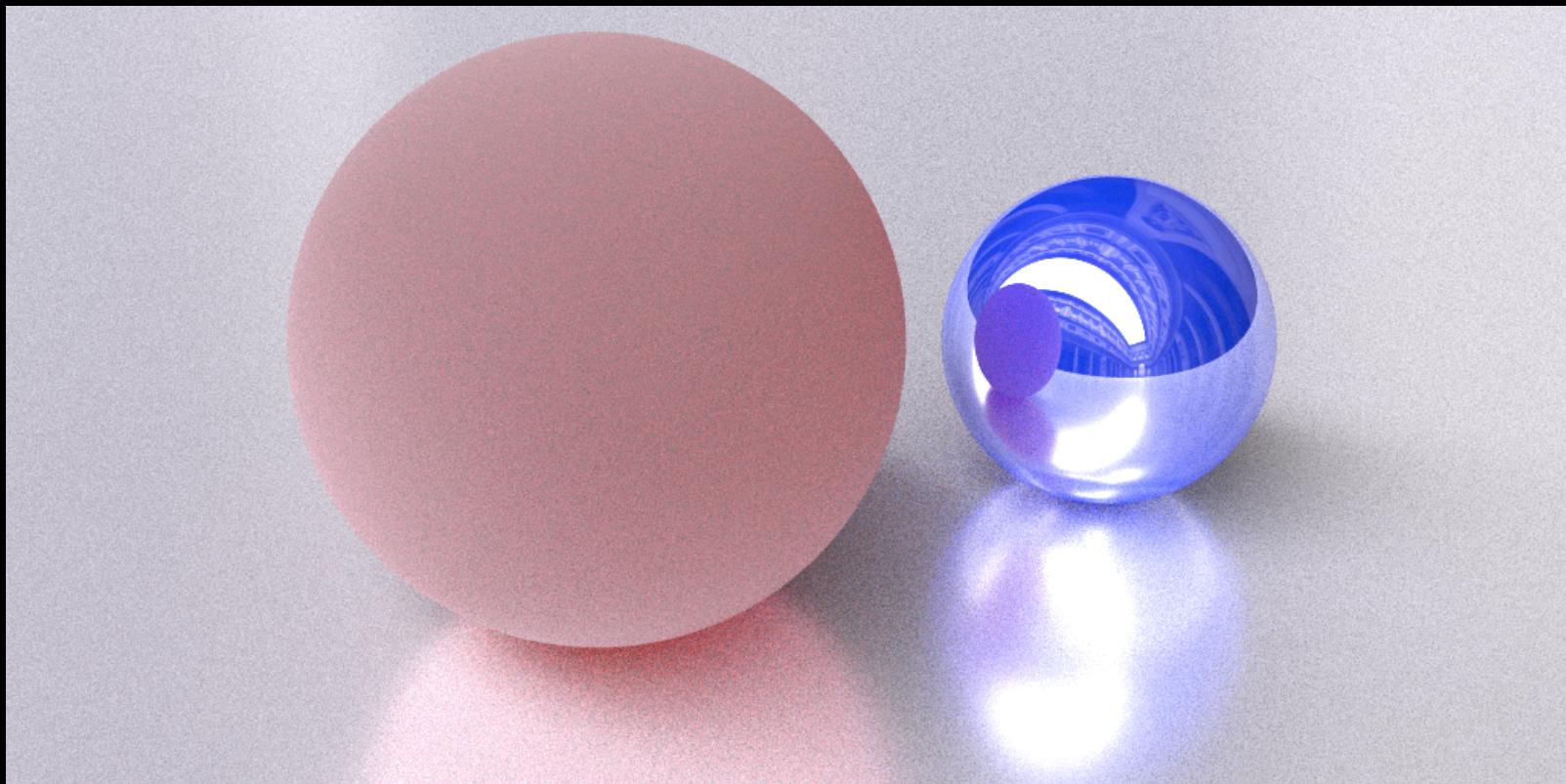
multiple bounces  
32 seconds



glossy reflection  
258 seconds



translucency  
681 seconds



environment  
776 seconds

and it only gets worse from here

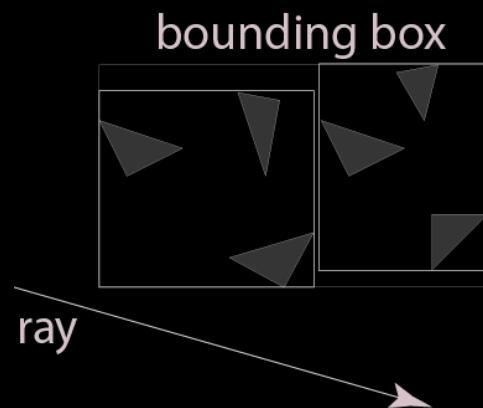
adding more realism involves computing more light bounces

the weird

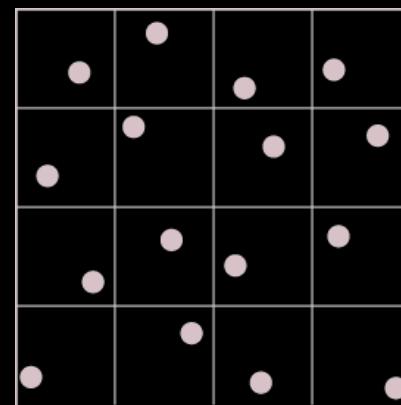
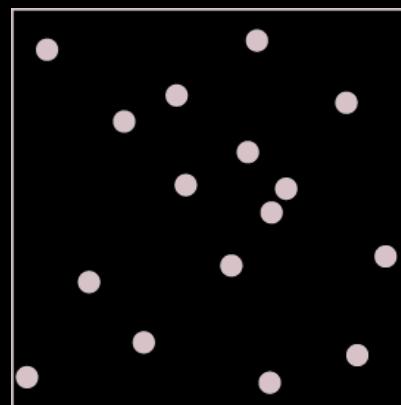
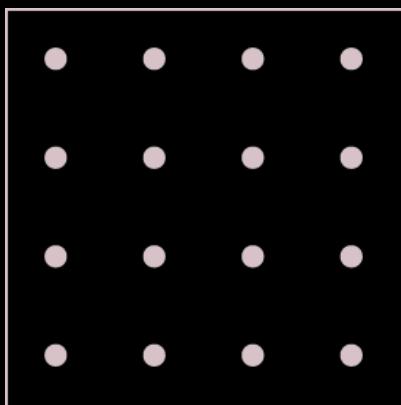
we could use approximations for the rendering equation  
but the final image will not be accurate

the following are a few different ways  
to improve render times  
*without changing final image*

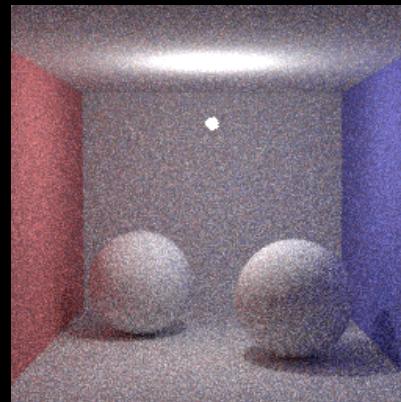
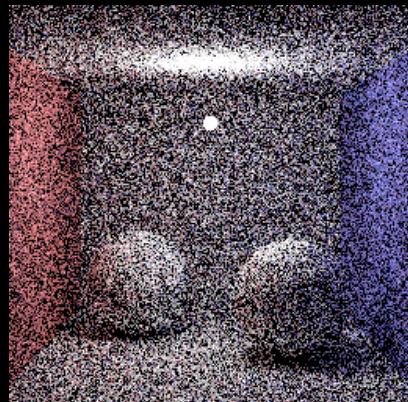
put triangles into boxes



throw darts



roll multiple unfair dice



[merity et al. [link](#)]

the end

[the third and the seventh, by alex roman link]



[ eye piece, by chris jones link ]



W.I.P.

05/02/14

[www.chrisj.com.au](http://www.chrisj.com.au)



[ snappers facial rig, by snappers mocap link ]

[roundtrip, by david gruwier [link](#)]



[light stage, hawkins et al. [link](#)]

